Affine Matrix Ball Construction

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AMS Meeting #1120
Fargo, ND

April 17, 2016

An analogue of Robinson-Schensted Correspondence

$$W \hookrightarrow\!\!\!\!\twoheadrightarrow \Omega_{dom}$$

- Left-hand side = affine symmetric group

- Right-hand side

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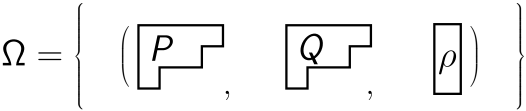

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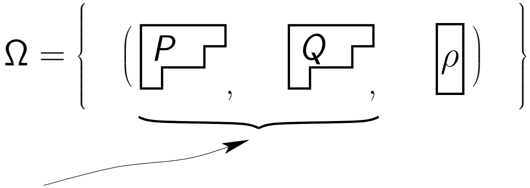

tabloids of same shape filled with $\overline{1} := 1 + n\mathbb{Z}, \overline{2}, \dots, \overline{n}$.

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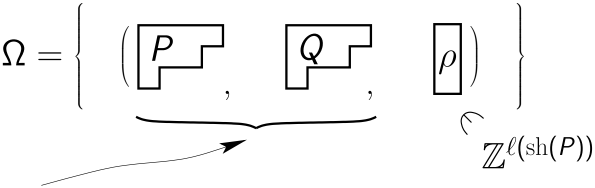

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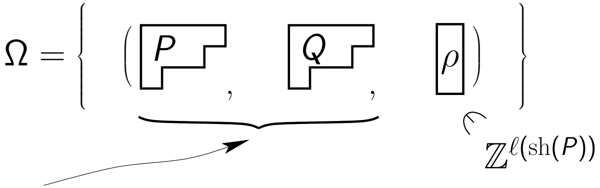

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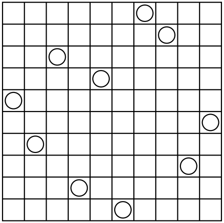
- Offset dominance: $\forall i, P, Q\ \exists r_i^{P,Q} \in \mathbb{Z} \cup \{-\infty\}$ s. t. $\rho_{i+1} \geqslant \rho_i + r_i^{P,Q}$

Matrix Ball Construction

w = 78351a2946



$P =$ $\qquad$ $Q =$

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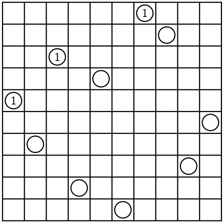

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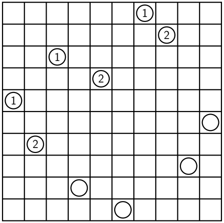

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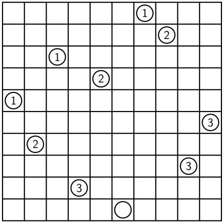

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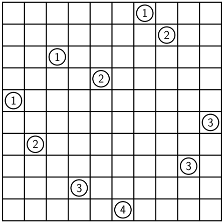

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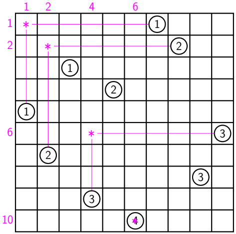

$P =$

1	2	4	6

$Q =$

1	2	6	10

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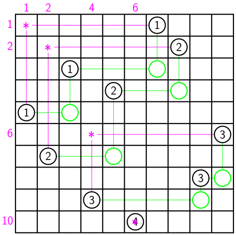

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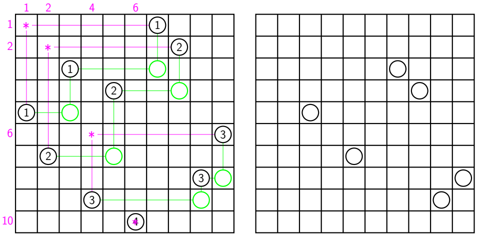

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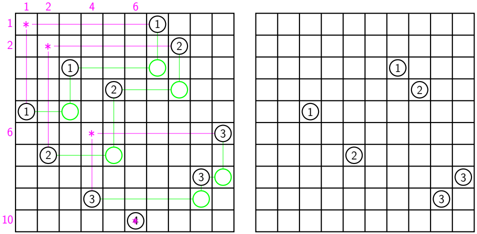

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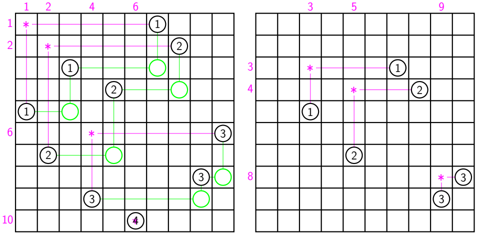

$P =$

1	2	4	6
3	5	9	

$Q =$

1	2	6	10
3	4	8	

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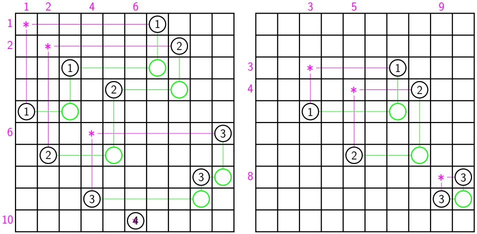

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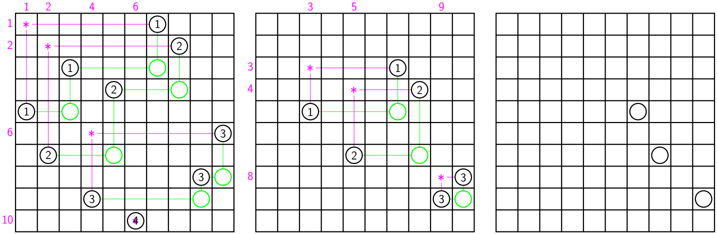

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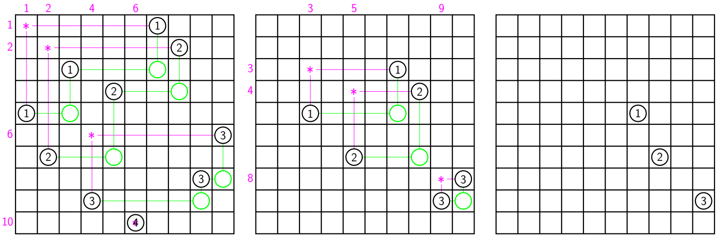

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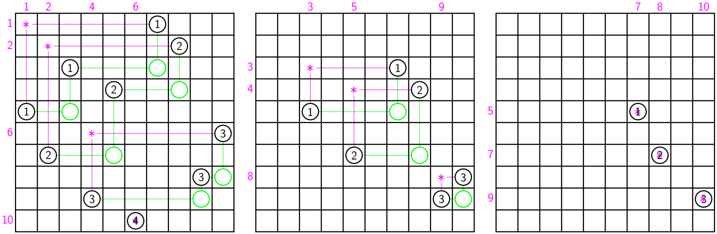

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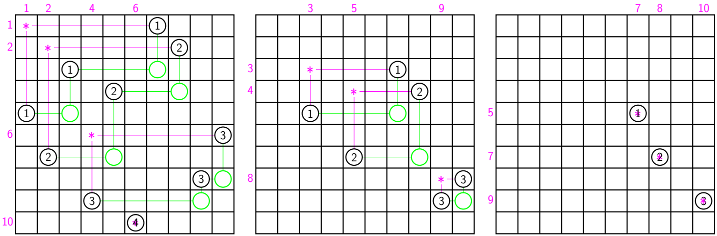

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Rmk. From tableaux can tell exact positions of $*$'s.

Proper numberings

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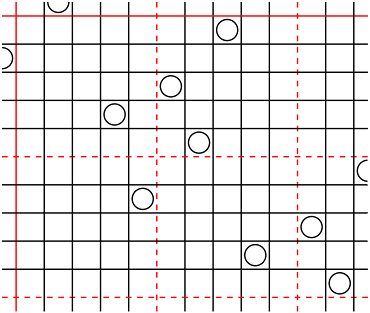

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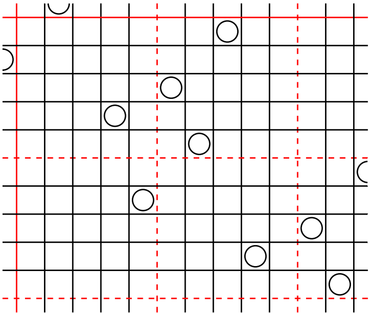

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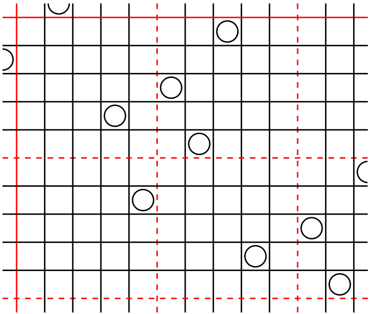

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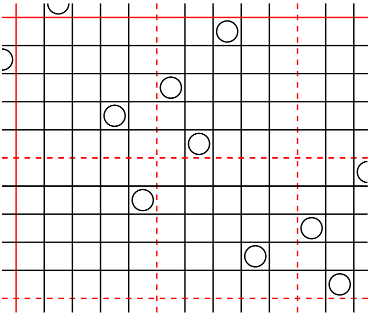

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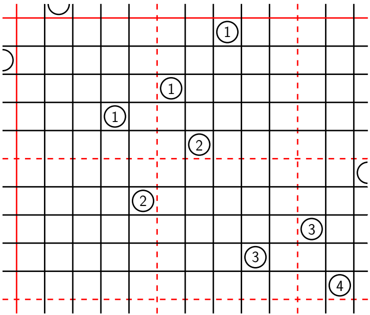

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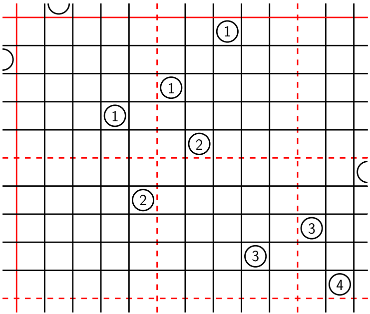

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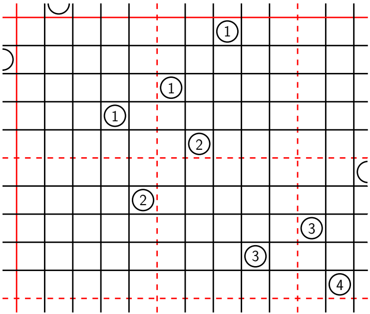

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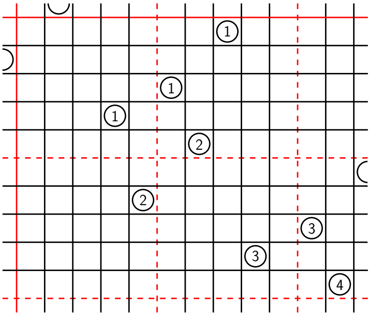

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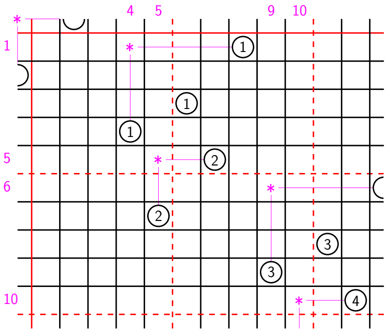

$P =$ | $\bar{4}$ | $\bar{5}$ |

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$\rho =$ | ? |

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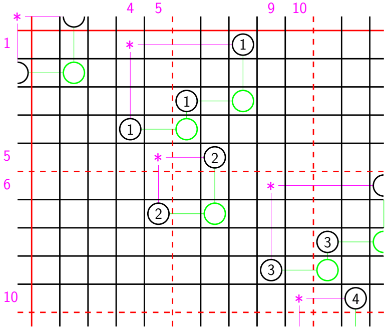

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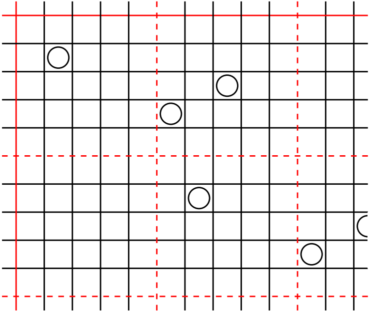

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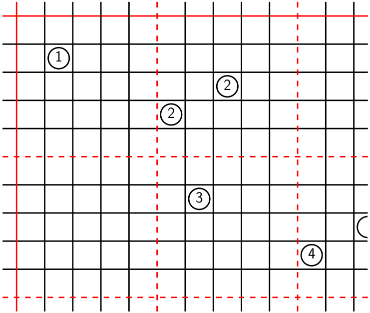

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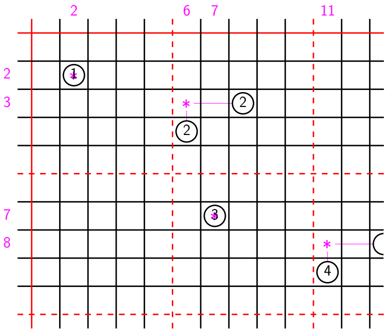

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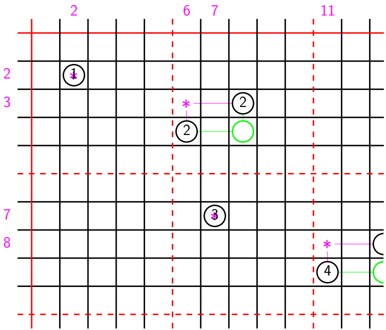

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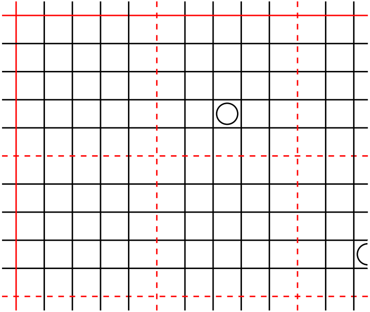

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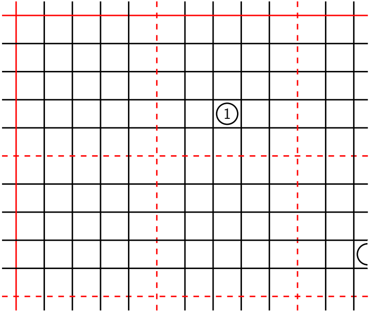

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- ***Thm.*** Proper numberings are periodic.
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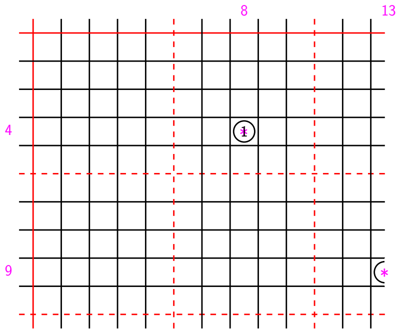

$$P = \begin{array}{|c|c|}\hline \overline{4} & \overline{5} \\ \hline \overline{1} & \overline{2} \\ \hline \overline{3} & \\ \cline{1-1} \end{array} \qquad Q = \begin{array}{|c|c|}\hline \overline{1} & \overline{5} \\ \hline \overline{2} & \overline{3} \\ \hline \overline{4} & \\ \cline{1-1} \end{array} \qquad \rho = \begin{array}{|c|}\hline ? \\ \hline ? \\ \hline ? \\ \hline \end{array}$$

Proper numberings

- $\mathcal{B}_w = \{\text{balls of } w\}$
- ***Def.*** $d : \mathcal{B}_w \to \mathbb{Z}$ is *proper* if
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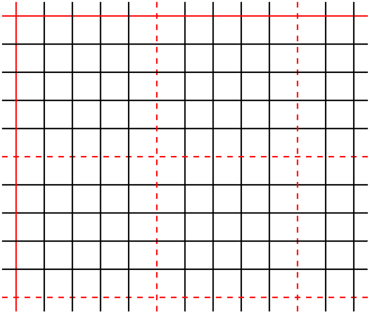

$P =$

$\overline{4}$	$\overline{5}$
$\overline{1}$	$\overline{2}$
$\overline{3}$	

$Q =$

$\overline{1}$	$\overline{5}$
$\overline{2}$	$\overline{3}$
$\overline{4}$	

$\rho =$

?
?
?

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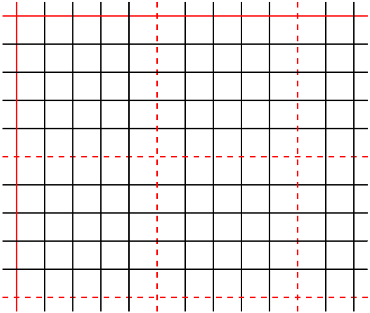

$P =$

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$\overline{3}$	

$Q =$

$\overline{1}$	$\overline{5}$
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$\overline{4}$	

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?
?
?

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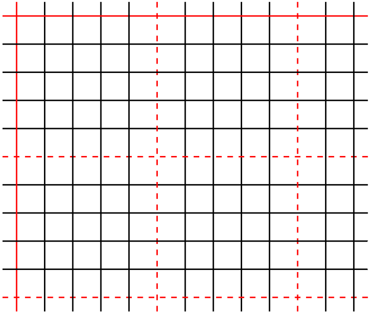

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$\overline{4}$	$\overline{5}$
$\overline{1}$	$\overline{2}$
$\overline{3}$	

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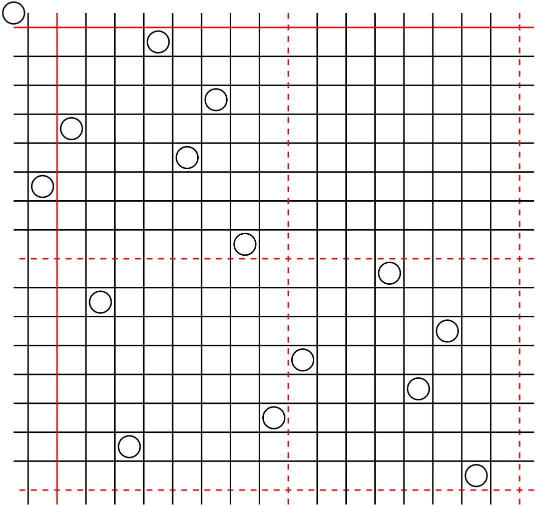
$\overline{2}$	$\overline{3}$
$\overline{4}$	

$\rho =$

0
1
1

Channel numberings

- ***Def.*** Channel = densest periodic collection of balls in negative slope



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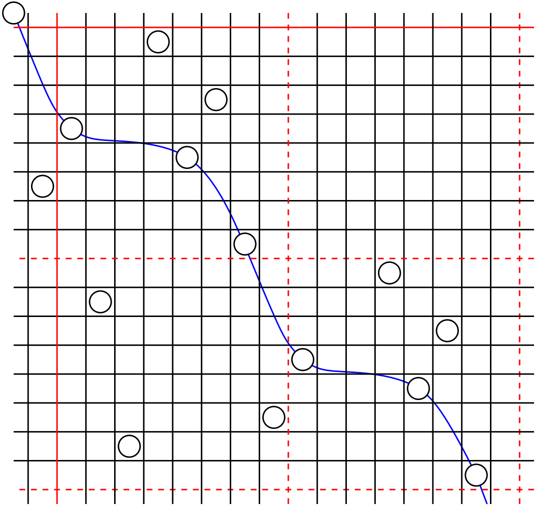

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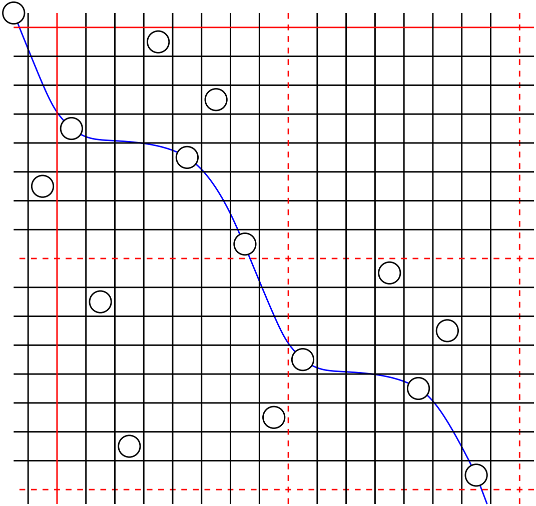

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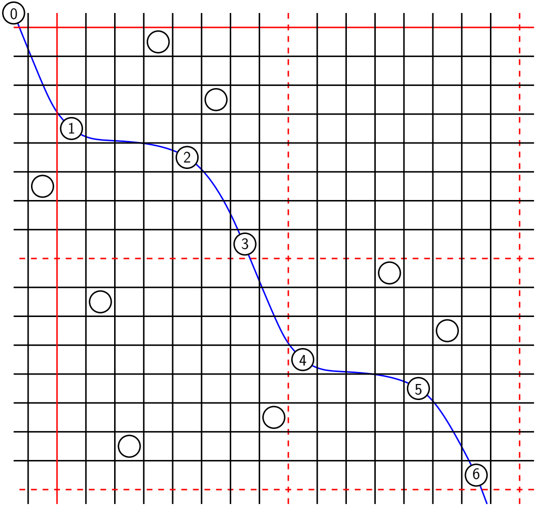

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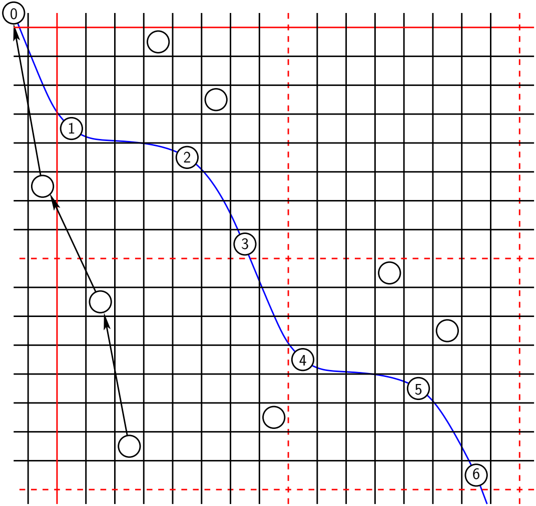

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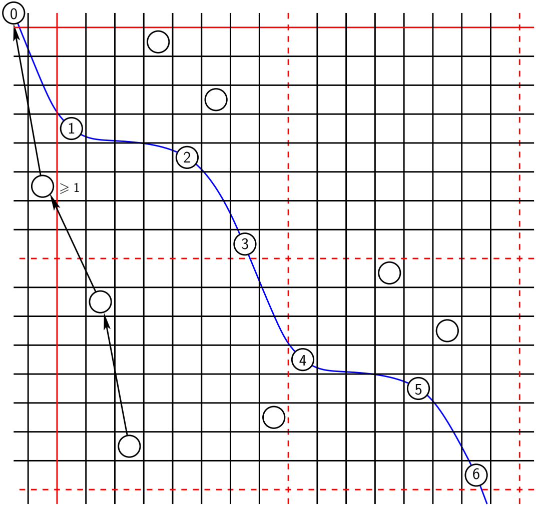

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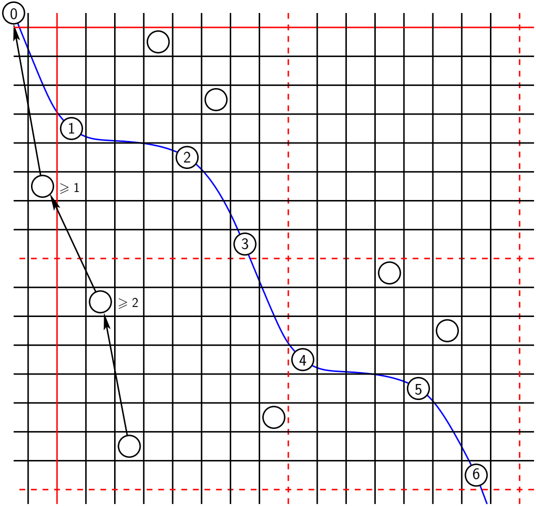

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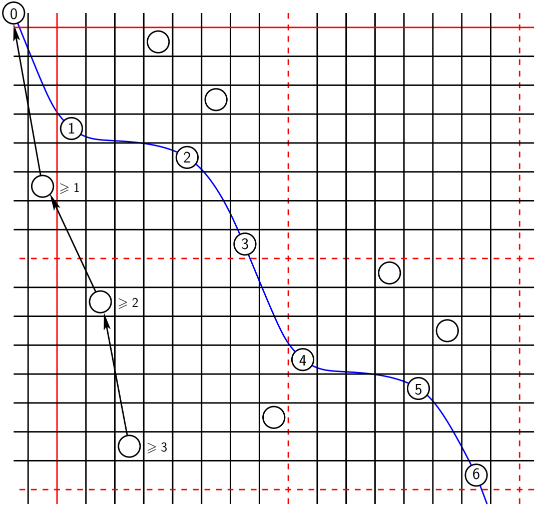

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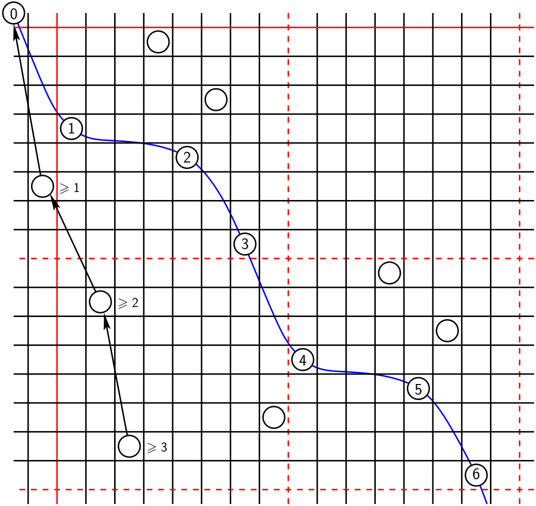

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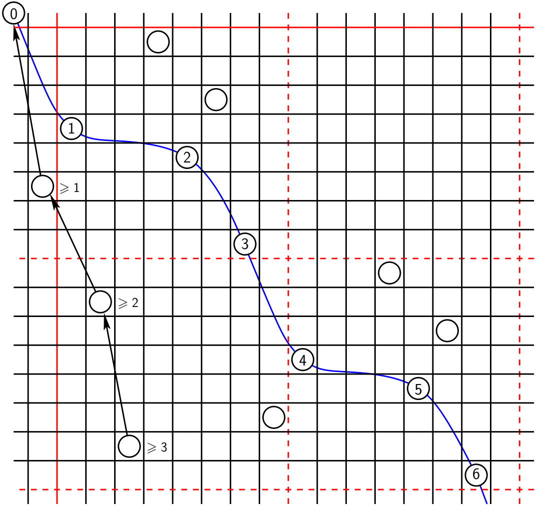

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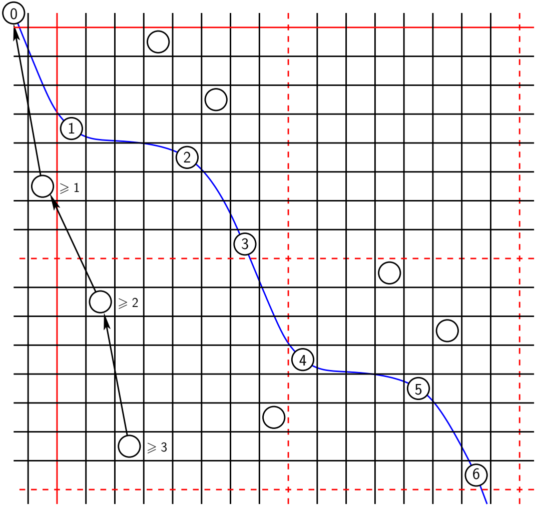

Motivation and History

(W, S) – Coxeter system, ground ring: $\mathbb{Z}[q^{\pm 1/2}]$

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  - Honeywill (2005): extension to bijection (not similar to RS)

Thank you!