Math 1271-002 (Calculus 1). Spring 2015.

Midterm Exam 1

Name:_____

1. **Do not open this exam until you are told to do so.**
2. This exam has 6 pages including this cover. There are 5 problems.
3. Not all problems are of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
5. Show an appropriate amount of work for each problem.
6. You may (but do not need to) use a scientific calculator.
7. No notecards are allowed.
8. **No cell phones, smartphones, headphones, or other devices allowed.**

1	
2	
3	
4	
5	
Total	

Problem 1. a. (2 points) Write down the definition of the derivative of a function f at a point a.

b. (3 points) Suppose $f(s) = \ln(\arcsin(s))$. Write down the definition of $f'(0.75)$. Your answer should not involve the symbol f or the symbol s. Do not attempt to evaluate the limit.

c. (5 points) Find the derivative of $f(s) = \frac{1}{\sqrt{s+1}}$ using the definition of derivative.

Answer: $f' =$ _____

Problem 2. (10 points) Find all the vertical and horizontal asymptotes of

$$f(x) = \frac{2x^2 + 4x - 6}{3x^2 - 9x + 6} + \arctan(x^3).$$

Answer:

Problem 3. Let c and k be two constants. Consider the piecewise-defined function

$$f(x) = \begin{cases} c \cdot e^x & x \geqslant 2 \\ kx + 3 & x < 2 \end{cases}$$

a. (3 points) Find the value of c for which the slope of the graph of $f(x)$ when $x = 3$ is 1.

b. (7 points) Find the values of c and k such that f is continuous and differentiable at all real numbers (DO NOT use the value of c found in part (a)).

Answer:

Problem 4. Consider the function $f(x) = 3x^2 + 4x + 1$.
a. (3 points) Find $f'(x)$.

Answer:

b. (3 points) Find the x-coordinate of the point where the tangent line is horizontal.

Answer:

c. (4 points) Find the equation of the tangent line at $(1, 8)$.

Answer:

Problem 5. (10 points) A function g defined for all real numbers has the following properties:

1. g is differentiable for $-1 \leqslant x < 4$.
2. $g'(x) \leqslant 0$ for $-1 \leqslant x < 4$.
3. $g''(x) > 0$ for $2 < x < 4$.
4. $g(4) = 2$.
5. $\lim_{x \to 4} g(x) = 0$.
6. g is continuous at $x = 5$ but not differentiable at $x = 5$.
7. $g'(0) = 0$.

On the axes below, draw a possible sketch of $y = g(x)$ on the domain $-1 \leqslant x \leqslant 6$.