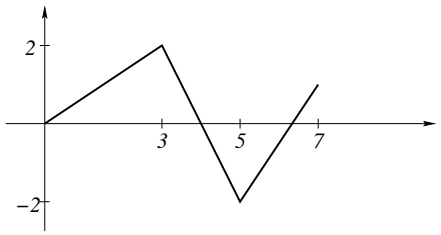
Math 1271-002 (Calculus 1). Spring 2015.

Practice Midterm 2 answers

Problem 1. (10 points) Given below are the graph of a function $f(x)$ and the table for a function $g(x)$.



x	0	1	2	3	4
$g(x)$	4	3	1	2	$\frac{20}{3}$
$g'(x)$	-2	$-\frac{5}{2}$	$\frac{1}{2}$	3	$-\frac{1}{3}$

Calculate the following. (NPC)

a. (2 points) $\frac{d}{dx}\left(f(g(x))\right)$ when $x = 0$.
Answer: 4

b. (2 points) $\frac{d}{dx}\left(g(f(x))\right)$ when $x = 3$.
Answer: Undefined

c. (2 points) $\frac{d}{dx}\left(f(x)g(x)\right)$ when $x = 2$.
Answer: 4/3

d. (2 points) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$ when $x = 4$.
Answer: -3/10

e. (2 points) $\frac{d}{dx}\left(f^{-1}(x)\right)$ when $x = 1$.
Answer: 3/2

Problem 2. (10 points) A curve in the plane is given by the equation $\sin(\pi xy) = y - 1$.

a. (2 points) Check that the point $(1, 1)$ lies on this curve.
$\sin(\pi \cdot 1 \cdot 1) = 0 = 1 - 1$
b. (8 points) Find the equation of the tangent line to the curve at $(1, 1)$.
Answer: $y - 1 = -\frac{\pi}{1+\pi}(x - 1)$

Problem 3. Let $y = ax^b \ln(x)$, for some nonzero constants a, b.

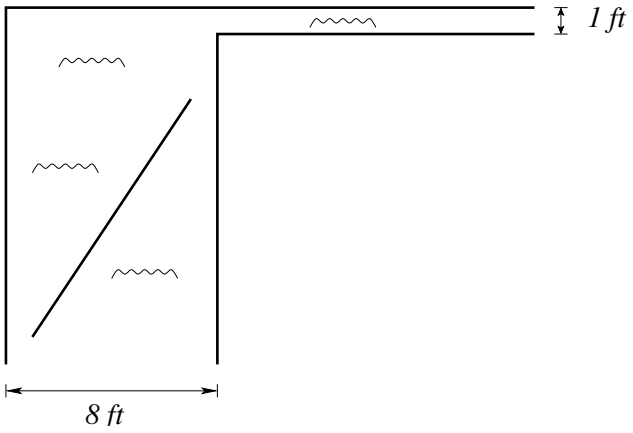
a. (3 points) Calculate $\frac{dy}{dx}$ in terms of a and b.
Answer: $\frac{dy}{dx} = ax^{b-1}(b\ln(x) + 1)$

b. (7 points) Find the values of a and b for which the function has a local maximum at $(e, 1)$. Check that the function indeed has a local maximum at $(e, 1)$.
Answer: $a = e$ $\qquad$ $b = -1$

Problem 4. A person walks out of point A at noon and heads north at a speed of 1 mph. Three hours later, another person runs out of A and heads east at 6 mph. At what rate is the distance between the two people increasing at 4pm?

Answer: $\frac{40}{\sqrt{52}}$

Problem 5. A wooden rod is floating toward a bend in the channel shown below. What is the maximal possible length of the rod that will get through?



Answer: $5\sqrt{5}$ft