Math 5705 (Enumerative Combinatorics); Fall 2016

Worksheet 21 (12/12/16)

Problem 1. *Suppose* $a_0 = 0$ *and* $a_n = 3na_{n-1} + 4n!$ *for* $n \geqslant 1$. *Find an appropriate generating function for the sequence* $\{a_n\}_{n\geqslant 0}$.

Problem 2. *The group of rotations of the cube has* 24 *elements. What are they? What is the cycle indicator of the action of this group on the vertices? What about on the faces?*

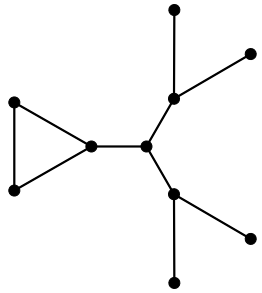
Problem 3. *Find an appropriate generating function for the number of labeled graphs with* n *vertices, all of which have degrees* 1 *or* 2.

Problem 4. *A group of people will make Christmas Tree ornaments. Some will make paper snowflakes while others will make felt ornaments. The group making snowflakes will split into three groups based on star design; moreover each person get to pick one of 5 paper colors. In the group making felt ornaments, each person gets to choose one of four designs. Let* $f(n)$ *denote the number of ways to achieve this when the total group size is* n *people. Find an appropriate generating function of the sequence* $\{f(n)\}_{n=0}^{\infty}$.

Problem 5. *Let* $h(n)$ *be the number of ways to choose a permutation of* n*, then for each cycle* C *either color each element of* C *red or blue, or else don't color any element of* C. *For instance,* $h(2) = 14$*, with nine possibilities for the identity permutation and five for the other. Set* $h(0) = 1$. *Find a simple expression for the exponential generating function for* $h(n)$.

Problem 6. *What is the radius of convergence of the power series* $\tan(3x)$*? Give an asymptotic upper bound for the coefficients of the power series.*

Problem 7. *What is the cycle indicator of the automorphism group of the graph shown?*



How many inequivalent (up to automorphism) colorings are there in 3 colors?