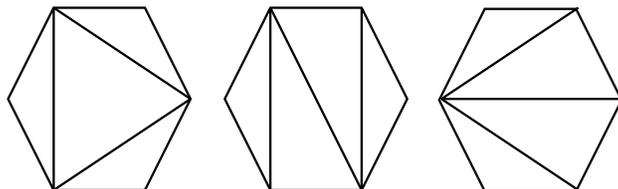


# Math 5705 (Enumerative Combinatorics). Fall 2017.

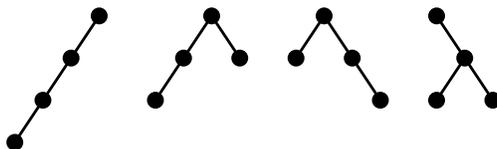
*Excursion into Cataland*  
*Version for Hosts of the first kind*

Below are some sequences of collections of objects.

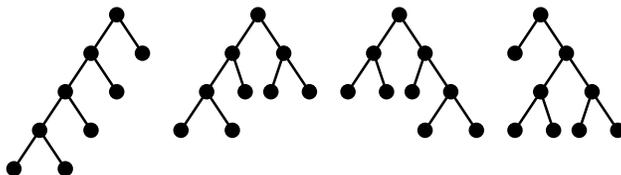
A *Triangulations* of an  $(n + 2)$ -gon by diagonals. Below are some examples of triangulations of a hexagon.



B *Binary trees* with  $n$  vertices are constructed as follows. Start with a root node at the top; from it you can move down and left to another vertex (“child”), or down and right to another vertex (or both or neither). At each of the two possible vertices you find another binary tree. Below are some of the binary trees with 4 vertices.



C *Complete binary trees* with  $2n + 1$  vertices are binary trees with  $2n + 1$  vertices where every node has either both children or none. Below are some of the complete binary trees with 9 vertices.



**Problem 1.** *How many of each of the above object are there for  $n = 1, 2, 3$ ?*

**Problem 2.** *Find a bijection between the set from B and the set from C.*

- Once everyone in the class is done with Problem 2, Travelers should move to the next group with respect to the outward orientation of the Earth’s surface (aka counter-clockwise, aka in the positive direction).
- The Travelers should describe the two object sequences they have on their sheet and their Problem 2 while the Hosts record it.
- The Hosts should describe the three object sequences they have on their sheet and their Problem 2 while the Travelers record it.
- Upon completion, the groups should work on Problem 3 from the Hosts’ sheet.

**Problem 3.** *Find a bijection between the set from A and the set from B.*

Once everyone in the class is done with Problem 3, Travelers should move to the next group and a discussion of Problems 3 should follow.