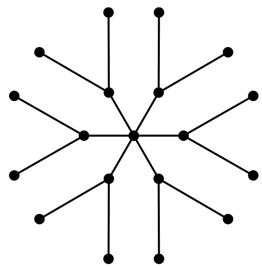
Math 5705 (Enumerative Combinatorics). Fall 2017.

Worksheet 19 (12/11/17)

Problem 1. *What is the average number of fixed points for the automorphism group of the graph shown. Note: this is not meant to be a computationally intensive problem.*



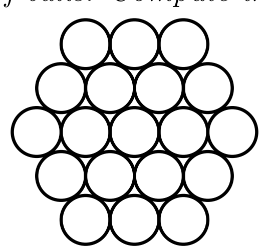
Problem 2. *Let g_n be the number of ways to choose a permutation of n and pick a distinguished cycle. For example, $g_2 = 3$ since there are two choices of distinguished cycle for the identity permutation and one choice of distinguished cycle for the transposition. Prove combinatorially that the numbers g_n satisfy the recurrence relation*

$$g_0 = 0, \qquad g_n = g_{n-1} + (n-1)! + (n-1)g_{n-1}, \text{ for } n \geqslant 1.$$

Use the recurrence relation to find the exponential generating function of the sequence $\{g_n\}_{n=0}^{\infty}$.

Problem 3. *There are n (distinguishable) fish in an aquarium. The person who feeds the fish gives an odd number of the fish either a red pollywog or a blue pollywog to eat. He gives an odd number of the fish either a black sea anemone, a purple sea anemone, or a green sea anemone to eat. The poor remaining fish get nothing to eat. (No fish gets more than one item.) For instance, $f(1) = 0$ (since there must be at least one fish that gets a pollywog and at least one fish that gets a sea anemone) and $f(2) = 12$ (two choices for which fish gets a pollywog, two choices for the pollywog color, and three choices for the sea anemone color). Find a simple formula for a suitable generating function of $\{f(n)\}_{n=0}^{\infty}$.*

Problem 4. *Consider the shown arrangement of balls. The group of the 12 symmetries of the hexagon acts on this collection of balls. Compute the cycle indicator of this this action.*



Problem 5. *Let $h(n)$ be the number of ways to choose a permutation of n, then color each cycle red or blue. For example, $h(2) = 6$, with four possible colorings of the identity permutation and two colorings for the transposition. Set $h(0) = 1$. Find a simple expression for the exponential generating function for $\{h(n)\}_{n=0}^{\infty}$.*

Problem 6. *Let $f(x) = \frac{1}{x^2-2x+35}$.*

(1) *What is the radius of convergence of the power series representing $f(x)$?*
(2) *Give an asymptotic upper bound for the coefficients of the power series.*